Bubbles, Banks and Financial Stability

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Abstract

This paper shows that the macroeconomic impact of rational bubbles in a limited commitment economy crucially depends on whether banks or ordinary savers hold the bubble. Banks hold the bubble asset when their leverage is high, when long-term real interest rates are low or when lax supervision allows them to enjoy high deposit insurance subsidies.

When banks are the bubble-holders, this amplifies the output boom by reducing loan-deposit rate spreads while the bubble survives but also deepens the recession when the bubble bursts. In contrast, the real impact of bubbles held by ordinary savers is more muted.

Keywords: Rational Bubbles, Banks, Credit Frictions

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1 Introduction

This paper studies the interaction of asset price bubbles, the banking system and the real economy in a general equilibrium model with credit constraints. The current financial crisis is the most immediate motivation for studying this interaction. Losses from subprime mortgage defaults depleted bank capital positions and led to a severe credit crunch and the deepest recession since the 1930s. This episode is not unique in history. In Japan, during the 1980s, real estate prices grew rapidly, allowing property developers to borrow from banks using real estate as collateral. When property prices collapsed in the 1990s, Japan went through a protracted banking crisis and a ‘lost decade’ of low economic growth. In all of these historical episodes, the banking system was exposed to a bursting housing bubble through a higher incidence of household default. As Reinhart and Rogoff (2008) show, using their rich time-series and cross-country data set, the resulting insolvency of the financial system leads to a deep downturn in real activity.

The one bubble episode that stands in clear contrast to all the costly boom-bust cycles described above is the 1998-2000 ‘technology bubble’. Stock prices rose and then collapsed dramatically without triggering a banking crisis or a deep recession. We want to understand why this bubble episode was so different from other recent bubbles. We build a model which focuses on bank exposures to the asset price collapse as an important reason why some busts lead to a banking crisis while others do not.¹ We then use the model to think about the factors that may have driven the banking system to become highly exposed to the subprime bubble in the 2003-2006 period.

Our framework is based on rational bubbles. Similarly to other recent papers in this literature (Caballero and Krishnamurthy (2006), Kocherlakota (2009), Ventura (2012), Martin and Ventura (2012), Farhi and Tirole (2012)), credit frictions lead to a shortage of means of saving and the use of dynamically inefficient production technologies.² This creates the conditions for bubbles to circulate and expand economic activity by reducing the severity of credit constraints.

Our innovation relative to the rest of the rational bubbles literature is to model financial intermediation explicitly. This realistic feature of our environment has the implication, overlooked by the rest of the literature, that asset price bubbles can be held by a variety of agents with different

¹Martin and Ventura (2012) argue that a reason why the ‘dot com’ bust did not lead to a recession is that it was immediately followed by the housing bubble which helped to expand the economy. This argument is plausible and we see it as complementary to our channel which stresses the health of the banking system.
²Other recent contributions include Hirano and Yanagawa (2010), Arce and Lopez-Salido (2011), Miao and Wang (2011) and Basco (2014).
economic roles. Our paper shows that the real effect of bubbles crucially depends on who holds them in equilibrium. Bubbles held by banks lead to a larger boom-bust cycle in credit and output compared to bubbles held by ordinary savers.

Bank bubbles have larger real effects due to the way the identity of the bubble holder affects the wealth distribution in equilibrium. Bubbles are risky assets which deliver a higher return compared to safe assets in order to compensate investors for losses in the bust. The bubble holder experiences strong growth of net worth while the bubble survives and then a sharp fall when the bubble bursts. When capital-constrained intermediaries hold the bubble, they expand credit supply and reduce lending spreads during the boom. In the crash, banks make large losses and inflict a credit crunch on the rest of the economy. This ‘credit supply effect’ of the bubble is unique to our model and it contributes significantly to the bubble’s impact on the real economy. In contrast, ordinary savers are credit unconstrained and their net worth does not play the same role in the credit intermediation mechanism. When savers hold the bubble, their wealth also fluctuates but this has a very limited impact on the credit constraints faced by other agents in the economy.

The paper also studies the conditions under which banks buy the bubble asset in equilibrium. This is in many respects the most important policy question of the paper. Why did banks choose to remain exposed to the collapse of the subprime bubble despite the possibility to shed this risk via securitisation markets?

Interestingly, in the baseline calibrated version of the model without government intervention, savers are the natural holders of bubbles. This is because their opportunity cost of funds is the deposit rate, while banks’ opportunity cost is the loan rate which is in general higher. Banks start to buy the bubble under a number of conditions which fit well the experiences during historical boom-bust cycles. First of all, lax supervision under the presence of deposit insurance creates a risk-taking subsidy that gives banks an advantage in holding the bubbly assets. Secondly, high bank leverage plays an important role by expanding credit supply and reducing the spread between lending and deposit rates. A lower wedge between the opportunity cost of funds for banks and savers results in higher financial institutions’ exposure to the bubble. Thirdly, low real interest rates expand the size of the bubble and, as a result, more of it ends up on bank balance sheets.

Our paper supports the view that well supervised and highly profitable banks should not be ex-

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An independent recent work by Miao and Wang (2012) constructs a model of rational bubbles with a banking sector. However, they do not analyse the implications of different bubble ownership.
posed to asset bubbles. The model suggests a number of reasons why the exposure of the banking system to the subprime bubble was large. The creation of complex structured products facilitated regulatory arbitrage (Acharya et al. (2013)), making effective supervision more difficult and increasing banks’ ability to leverage up and hold risky assets. In addition, the rapid growth of the broker-dealer sector in the 2000s increased the importance of these lightly regulated and highly leveraged entities in the financial system. In this paper, we implement these important developments through movements in parameters that govern bank leverage and supervision quality. We show that these developments, as well as the well-documented decline in real interest rates, can lead to large banking system exposure to the bubble asset and a more amplified boom-bust cycle for the real economy.

Our paper’s findings are in line with several important stylised facts uncovered by the literature on the empirical regularities around financial crises. Our focus on rational bubbles that can only occur in a low interest rate environment is supported by Jorda et al. (2012) who report that growth adjusted real interest rates are very low for several years in the run up to global crises.

Our finding that bank exposure amplifies the real effects of asset price bubbles is supported by the work of Claessens et al. (2011) and Schularick and Taylor (2012) who present international evidence that credit booms are associated with larger cumulative increases and steeper subsequent declines in GDP. This implication of our model is also supported by the microeconomic evidence in Mian and Sufi (2010). In addition, our focus on bank bubble exposures is consistent with the evidence in Claessens et al. (2011) who argue that housing boom-bust cycles lead to larger output fluctuations compared to equity cycles. Since housing finance is mostly bank-intermediated and occupies a large fraction of bank balance sheets, bank bubble exposures are more likely to arise during booms and busts in the housing market than during equity boom-bust episodes.

Finally, our model also has the novel implication that credit spreads are low during bubbly episodes and then rise sharply as the bubble collapses. This theoretical prediction is in line with the empirical evidence in Gilchrist and Zakrajsek (2012) who show that bond risk premia are strongly related to the health of financial institutions.

The rest of the paper is structured as follows. Section 2 introduces the economic environment, section 3 describes the main equilibrium conditions. Section 4 outlines the model calibration. Section 5 discusses the conditions for bubble existence and the determinants of who owns the
bubble in equilibrium before analysing the implications of bubble ownership for the real effects of bubbly episodes. Section 6 concludes.

2 The Model

The economy is populated with three kinds of agents. There is a continuum of infinitely lived entrepreneurs and a continuum of infinitely lived workers both of measure 1. There is also a continuum of bankers who have finite lives and can die stochastically. There is also the government which provides deposit insurance.

2.1 Entrepreneurs

The entrepreneur sector is based on Kiyotaki (1998). Each entrepreneur is endowed with a constant returns to scale production function which converts labor $h_t$ into output in the next period $y_{t+1}$.

$$y_{t+1} = a_i^t h_t,$$

(1)

where $a_i^t$ is a productivity parameter which is known at time $t$.

In each period some entrepreneurs are productive ($a_i^t = a^H$) and the others are unproductive ($a_i^t = a^L < a^H$). A productive entrepreneur in this period may become unproductive in the next period with probability $\delta$, and an unproductive entrepreneur in this period may become productive with probability $n\delta$. This probability is independent across entrepreneurs and over time. This Markov process implies that the fraction of productive entrepreneurs is stationary over time and equal to $n/(1 + n)$.

Entrepreneurs are ex-ante identical and have log utility over consumption streams (discounted at rate $\beta$)

$$U^E = E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t, \quad 0 < \beta < 1.$$

(2)

Entrepreneurs purchase consumption ($c_t$), bubbles ($m_t^E$) at price $\mu_t$ and bonds $b_t$. They also pay wages to the workers they hire $w_t h_t$ in order to receive future revenues $a^t h_t$ which the government taxes at rate $\tau_t$ after deducting debt repayments. $w_t$ and $h_t$ denote real wage and labor respectively.
The flow-of-funds constraint is given by

\[ c_t + w_t h_t + m^e_t \mu_t - b_t = (1 - \tau_t) \left( a^i h_{t-1} - R^d_{t-1} b_{t-1} + m^e_{t-1} \mu_t \right) \equiv (1 - \tau_t) z_t \quad (3) \]

where \( z_t \) stands for entrepreneur’s net worth. \( R^i_t \) is the interest rate which is equal to the loan rate \( R^l_t \) when the entrepreneur is a borrower and the deposit rate \( R^d_t \) when he is a saver. The bubble asset is a durable but intrinsically worthless asset which has no productive or consumption value.\(^4\)

Following Weil (1987) we consider risky bubbles which may collapse with probability \( 1 - \pi \) in every period.\(^5\) Let \( \tilde{\mu}_{t+1} \) be the stochastic realisation of the bubble value in the following period. It will follow a process given by:

\[ \tilde{\mu}_{t+1} = \begin{cases} 
\mu_{t+1} > 0 & \text{with probability } \pi \\
0 & \text{with probability } 1 - \pi 
\end{cases} \quad (4) \]

Due to limited commitment in the credit market, agents will only honour their promises if it is in their interests to do so. We assume that only a fraction \( \theta \) of the value of the entrepreneur’s assets (output and bubble holdings) can be seized by creditors. Hence the collateral constraint is given by:

\[ R^d_t b_t \leq \theta E_t \left( y_{t+1} + m^e_t \tilde{\mu}_{t+1} \right), \quad 0 < \theta < 1. \quad (5) \]

Entrepreneurs maximize (2) subject to (3) and (5).

### 2.2 Banks

We assume that only banks can enforce debt repayments in our economy. Consequently, all borrowing and lending will be bank-intermediated. Bankers are risk neutral and live for a stochastic length of time. Once bankers receive an ‘end of life’ shock, they liquidate all their asset holdings and consume their net worth before exiting.\(^6\) This shock hits with probability \( 1 - \gamma \).

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\(^4\)We assume that investors cannot short the bubble.

\(^5\)We assume this probability is constant over time. Also, we assume that once bubbles burst they never arise again.

\(^6\)In order to keep the population of bankers constant, we also assume that in each period measure \( 1 - \gamma \) of new bankers are born with small initial endowments (net worth). Since those small initial endowments do not affect the subsequent analysis, we do not analyze those explicitly.
Banks maximize the following objective function:

\[ U^B = E_0 \sum_{t=0}^{\infty} (\beta \gamma)^t c'_t \]  

subject to the following balance sheet constraint:

\[ c'_t + b_t + \mu_t m^b_t = n_t + d_t. \]  

In each period the bank uses its own net worth \((n_t)\) and deposits \((d_t)\) from the savers in order to lend \((b_t)\), purchase bubbles \((m^b_t)\) or consume \((c'_t)\). We assume that intermediation is entirely costless. We also assume limited liability. Then the evolution of net worth is given by

\[ n_{t+1} = \max \left( R^d_t b_t + \hat{\mu}_{t+1} m^b_t - R^d_t d_t, 0 \right) \]  

which is constrained to be non-negative due to limited liability. When the value of bank assets falls below the value of liabilities, the bank goes bankrupt.

We assume that bank deposits benefit from deposit insurance (hereafter DI). This captures both the explicit guarantees of small deposits in commercial banks but also the implicit guarantees due to the systemic importance of formally uninsured financial institutions. Due to government guarantees, savers have little incentive to monitor their banks and impose market discipline on their behaviour. This could give rise to two types of moral hazard in the absence of bank regulation.

First, following Gertler and Karadi (2011), the banker may divert (and consume) \(1 - \lambda\) fraction of deposits at the cost of losing the ability to operate a bank for ever. In Gertler and Karadi (2011) uninsured bank creditors restrict the deposits that the intermediary can issue according to the following incentive compatibility constraint:

\[ (1 - \lambda) d_t \leq V(n_t). \]  

The left hand side of equation (9) is the value when the banker diverts, while the right hand side is
the value when he does not (i.e., the continuation value of the bank), which is defined by

$$V(n_t) = \max_{\{c_{t+j}, b_{t+j}, d_{t+j}, m_{t+j}\}_{j=0}^{\infty}} E_t \sum_{j=0}^{\infty} (\beta \gamma)^j c'_{t+j}.$$

In our economy, the presence of DI means that (9) is imposed and enforced by bank regulators. Hereafter we refer to (9) as the bank’s borrowing constraint.

A second type of moral hazard problem arises because the bank can invest excessively in the bubble asset, going bankrupt when the bubble bursts and creating deposit insurance costs for taxpayers. Following Repullo and Suarez (2004), risk-taking banks will want to have as low a recovery value in bankruptcy as possible in order to maximise the insurance payout paid by the deposit insurance corporation—this is known as the DI subsidy. In section 3.2, we show that some banks will indeed choose to specialise in holding bubbles (we call these “bubbly banks”) while others will specialise in holding only safe loans (“lending banks”).

In principle, the borrowing constraint (9) could be tightened further in order also to discourage the creation of bubbly banks which are profitable only due to DI subsidies. For example, in Martinez-Miera and Suarez (2014) tighter capital requirements work exactly in this way. Such a policy does of course have the undesirable side-effect of also restricting lending for productive purposes. Instead, we assume that bank supervision exists in order to detect banks that are trying to undertake large exposures to risky bubbles.\(^7\) When successful, supervision can eliminate risk-taking through holding bubbles without crowding out entrepreneurial loans.

However, we assume that supervision is imperfect: banks that hold more than \(\xi\) share of bubbles on their balance sheet are caught for sure but those with a smaller share escape undetected. This means that the banks are subject to a supervisory constraint that limits their bubble holdings to a maximum share \(\xi\) of the bank’s balance sheet.

$$\mu_t m_{t}^b \leq \xi \left(b_t + \mu_t m_{t}^b\right).$$

(10)

A lower value of \(\xi\) in our setting stands for a supervisory regime which is more effective at preventing risk-taking by the banks. In the baseline calibrated version of the model, capital regulation and

\(^7\)We assume that supervisors only inspect the banks after the opportunity to divert funds has passed. Hence supervision cannot eliminate the ability of the bank to steal depositors’ money and the borrowing constraint (9) is still needed.
bank supervision will be effective at ensuring that bank default never occurs. We will then consider how a deterioration in bank supervision affects banks’ bubble holdings and financial fragility.

The bank maximizes (6) subject to (7), (8), (9) and (10).

### 2.3 Workers

Unlike the entrepreneurs, the workers do not have access to the production technology nor any collateralizable asset in order to borrow. They maximize the following utility

$$U^w = E_t \sum_{t=0}^{\infty} \beta^t \left( c^w_t - \frac{h_t^{1+\eta}}{1+\eta} \right)$$

subject to their flow-of-funds constraint

$$c^w_t + m^w_t \mu_t - b^w_t = w_t h_t + m^w_{t-1} \mu_t - R^d_{t-1} b^w_{t-1},$$

where superscript ‘w’ denotes workers’ choices.

### 2.4 The Government

The government regulates banks and levies taxes on entrepreneurs in order to pay out the deposits of failing banks. We assume that the government follows a balanced budget rule. Taxes are only levied whenever bailout spending is necessary. For the rest of the time, taxation is zero.

### 3 Equilibrium

#### 3.1 Entrepreneurs’ optimal behavior

The entrepreneurs’ problem can be interpreted as a savings problem with uncertain returns. Since their period utility function is logarithmic and there is no labour income or transfer income, entrepreneurs consume a constant fraction of net worth ($z_t$) and save the remaining $\beta$ fraction:

$$c_t = (1 - \beta) z_t.$$  \hspace{1cm} (13)

An entrepreneur with productivity $a_t = a^H \cdot a^L$ has several possibilities for accumulating net
worth. She can undertake unleveraged investments using her own technology earning an after-tax return of \((1 - \tau_{t+1})a_t/w_t\). She can deposit in the banking system, earning \((1 - \tau_{t+1})R^d_t\). She can pledge future output to banks and borrow at interest rate \(R^l_t\) and invest in her own production technology. When the borrowing constraint (5) binds, the after-tax rate of return on a leveraged productive investment is equal to

\[
(1 - \tau_{t+1}) \frac{a_t(1 - \theta)}{w_t - \theta a_t/R^d_t},
\]

(14)

Finally, the entrepreneur can invest in bubbles whose rate of return is given by \((1 - \tau_{t+1})\tilde{\mu}_{t+1}/\mu_t\).

The unproductive entrepreneurs have relatively poor investment opportunities and therefore they are savers (depositors) in equilibrium. Since both deposits and production are riskless

\[
R^d_t = \frac{a^L_t}{w_t}
\]

(15)

whenever low productivity agents are active in production. This will happen when the credit constraints on banks and borrowing entrepreneurs are tight enough and the productive entrepreneurs cannot absorb all national saving. If

\[
R^d_t > \frac{a^L_t}{w_t},
\]

(16)

then savers do not produce.

Bubbles are risky. When the saver holds the bubble, the arbitrage condition for bubbles is determined by the saver’s state-contingent wealth valuation

\[
E_t \left[ \frac{1}{c^L_{t+1}} (1 - \tau_{t+1}) \tilde{\mu}_{t+1} \right] = E_t \left[ \frac{1}{c^L_{t+1}} (1 - \tau_{t+1}) \right] R^d_t,
\]

(17)

where \(1/c^L_{t+1}\) is the shadow value of wealth at time \(t + 1\) of an entrepreneur who is unproductive at time \(t\).\(^8\) The expectation operator is taken over whether the bubble survives or crashes.

High productivity entrepreneurs enjoy better returns on production so they are the ones who borrow in equilibrium. From equation (14), when \(a^H_t/w_t > R^d_t\), their borrowing constraints bind,\(^8\)

\[
1/c^L_{t+1} = (1 - \beta)Z^L_{t+1}
\]

where \(Z^L_{t+1}\) is the net worth of low-productivity entrepreneurs. Its aggregate revolution is derived later (equation (46)).
and the rate of return on a leveraged productive investment is equal to

\[(1 - \tau_{t+1}) \frac{a^H(1 - \theta)}{w_t - \theta a^H / R_l} > \frac{a^H}{w_t} > R_l.\]  

(18)

High productivity entrepreneurs can also purchase the bubble asset, although in equilibrium, it turns out that they do not do so. The intuition for this result is that, due to the arbitrage condition for savers (17), the bubble yields a return that is closely tied to the deposit rate. This is much lower than the leveraged rate of return for productive agents (18) thus making bubble ownership unattractive for them. Furthermore, as long as \(\theta < 1\), using the bubble asset as collateral is also not attractive because the borrowing constraint forces the leveraged holder to use some of its own funds in purchasing the bubble. This is costly for productive agents because it reduces the amount they can invest in their very high yielding productive projects. In subsequent sections, we first guess that productive agents hold no bubbles and verify it in any numerical solutions of the model.

The unwillingness of productive agents to hold the bubble is in contrast to the outcome in Kocherlakota (2009). He considers a similar credit-constrained economy and constructs an equilibrium in which bubbles are used as collateral. The difference between his result and ours stems from different assumptions on the collateral constraint and on the nature of debt contracts.\(^9\)

When high productivity entrepreneurs only invest in leveraged productive projects, then from (3) and (5) their investment (employment) is:

\[h_t = \frac{\beta z_t}{w_t - a^H \theta / R_l}.\]  

(19)

The entrepreneur saves a \(\beta\) fraction of wealth \(z_t\) and uses her entire savings as a downpayment for wage payments to the workers she hires.

\(^9\)In Kocherlakota (2009) borrowers are indifferent between owning a bubble to use as collateral and selling their bubble holdings and producing without leverage. He assumes that the full value of bubbles is collateralizable and that debt contracts are contingent on the realisation of the bubble price. In our model, this corresponds to assuming state contingent debt contracts and \(\theta = 1\) for the bubble asset.

However, if the bubble is not fully collateralizable in his model (i.e. \(\theta < 1\) is assumed), productive agents strictly prefer to sell the bubbles rather than own them and use them as collateral. In addition, our assumption of uncontingent debt without the possibility of ex post default makes the strategy of borrowing using the bubble as collateral risky. This encourages risk-averse productive agents to sell their bubble holdings rather than keep them and use them as collateral.

In a robustness exercise, which is available upon request, we found that the riskiness of using bubbles as collateral ensured that productive agents’ bubble holdings are either zero or very small for a wide range of parameter values even when we assume that bubbles can be fully collateralised as Kocherlakota (2009) does.
3.2 Bankers’ optimal behaviour

As we discussed in Section 2.2, in each period a bank has the investment strategy of becoming either a lending bank or a bubbly bank. In this section, we first describe the bank’s optimal behavior in each case, and then determine the equilibrium investment strategy choice.

3.2.1 Lending banks

Let $V^l(n_t)$ be the value of a lending bank with net worth $n_t$ who chooses consumption ($c^t_t$) and loans ($b^t_t$). This can be represented in recursive form as follows:

$$V^l(n_t) = \max\{c^t_t, b^t_t\}$$

subject to the budget constraint (7) and the flow of funds constraint (8). This value is equal to current consumption and the expected future discounted value of net worth $\beta E_t[\gamma V(n_{t+1}) + (1 - \gamma) n_{t+1}]$ which includes the continuation value of being a banker if (s)he survives with probability $\gamma$. With probability $1 - \gamma$, the banker exits and consumes his/her entire net worth. We assume that it is possible to switch between being a bubbly or lending bank costlessly in every period. Therefore the continuation value of being a banker, $V(n_{t+1})$, is given by

$$V(n_{t+1}) = \max\{V^l(n_{t+1}), V^b(n_{t+1})\},$$

where $V^b(n_{t+1})$ is the value of the banker if (s)he chooses to set up a bubbly bank in period $t + 1$. We describe the behavior of bubbly banks in the next subsection.

Since the banker is risk neutral, we guess that the value function is linear in $n_t$

$$V^l(n_t) = \phi_t^l n_t, \quad (21)$$

and the continuation value is given by

$$V(n_{t+1}) = \phi^t_{t+1} n_{t+1},$$

$$\phi^t_{t+1} = \max(\phi^l_{t+1}, \phi^b_{t+1}).$$
When $\phi_t > 1$, the banker invests all his net worth in loans and only consumes when he exits.

The lending banker does not want to hold bubbles as long as:

$$
E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) \tilde{\mu}_{t+1} \right] < E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) \right] R_t^l,
$$

(22)

where $1 - \gamma + \gamma \phi_{t+1}$ is the pricing kernel of the banker. We will confirm that equation (22) holds in equilibrium.

In equilibrium, the borrowing constraint binds when $R_t^l > R_t^d$. Then deposits are given by:

$$
d_t^l = \frac{\phi_t^l}{1 - \lambda} n_t.
$$

(23)

Using (7) and (23) the return per unit of wealth is:$^{10}$

$$
u_{t+1}^l = R_t^l + (R_t^l - R_t^d) \frac{\phi_t^l}{1 - \lambda}.
$$

(24)

Combining (20), (21) and (24) we have a functional equation for $\phi_t^l$:

$$
\phi_t^l = \beta E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) u_{t+1}^l \right].
$$

(25)

### 3.2.2 Bubbly banks

The problem of the bubbly banks is very similar to that of the lending banks apart from the fact that they also hold bubbles. We guess that the value function of the bubbly bank at time $t$ is given by

$$
V^b(n_t) = \phi_t^b n_t.
$$

(26)

When the borrowing constraint binds, deposits are given by:

$$
d_t^b = \frac{\phi_t^b}{1 - \lambda} n_t.
$$

(27)

$^{10}$Because loans are safe we can ignore the max operator in (8).
When the borrowing constraint (23) and the regulatory constraint (10) bind, the stochastic rate of return on wealth from a bubbly bank, \( u^b_{t+1}(\tilde{\mu}_{t+1}) \), is given by:

\[
u^b_{t+1}(\tilde{\mu}_{t+1}) = \begin{cases} 
\frac{u^b_{t+1}|s}{1-\lambda} & \text{w.p. } \pi \\
\frac{u^b_{t+1}|c}{1-\lambda} & \text{w.p. } 1-\pi 
\end{cases}
\]

where \( u^b_{t+1}|s \) and \( u^b_{t+1}|c \) respectively denote the rate of return on wealth when the bubble survives \((\tilde{\mu}_{t+1} = \mu_{t+1})\) and crashes \((\tilde{\mu}_{t+1} = 0)\). If \( u^b_{t+1}|c = 0 \) holds, the bank compares the two rates of return only in the state in which the bubble survives. Furthermore, whenever the bubbly banks are active, the bubble rate of return conditional on survival must be higher than the loan rate:

\[
\frac{\mu_{t+1}}{\mu_t} > R^d_t.
\]

Then, the bubbly bank holds the bubble up to the regulatory limit. Hence the condition for the regulatory constraint (10) to bind is \( u^b_{t+1}|c = 0 \). Intuitively, the option to go bankrupt allows the bubbly bank to avoid some of the losses from the collapse of the bubble while being able to enjoy all the profits while the boom continues. In the absence of regulation, bubbly banks want to hold only bubbles.\(^{11}\) However, the regulatory constraint binds and limits their bubbles holdings to a \( \xi \) fraction of total assets.

Combining (20), (21) and (28) we can derive a functional equation for \( \phi^l_t \):

\[
\phi^b_t = \beta E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) u^b_{t+1} \right].
\]

### 3.2.3 Equilibrium investment strategy choice

The ability to switch costlessly in every period between being a bubbly and a lending bank ensures that, in equilibrium, the value of the two investment strategies are equalized at all times when both types operate. This is the case we focus on in subsequent sections. Therefore, we have

\[
\phi^l_t = \phi^b_t = \phi_t.
\]

\(^{11}\)This result was first shown by Repullo and Suarez (2004).
Then, from (25) and (30) we obtain:

\[ E_t \left[ (1 - \gamma + \gamma \phi_{t+1}) \left( u_{t+1}^b - u_{t+1}^l \right) \right] = 0. \]  \hspace{1cm} (32)

In equilibrium, the fraction of bubbly banks and lending banks is determined to satisfy equation (32). When (32) is strictly negative, bubbly banks do not operate\(^\text{12}\).

Combining (20), (21), (23) and the bank’s balance sheet (7) we can derive a functional equation for \( \phi_t \):

\[ \phi_t = \frac{\beta E_t \left[ (1 - \gamma + \gamma \phi_{t+1}) R^l_t \right]}{1 - \beta E_t \left[ (1 - \gamma + \gamma \phi_{t+1}) \frac{R^l_t - R^d_t}{1 - \lambda} \right]}. \]  \hspace{1cm} (33)

The banker makes a leveraged investment in entrepreneurial loans. (33) reflects the value per unit of net worth of having such an opportunity: it is equal to the return on the bank’s loan book (the numerator), suitably boosted by leverage (the denominator).\(^\text{13}\)

### 3.2.4 Supervision, deposit insurance and the risk-taking subsidy

The ability of bubbly banks to go bankrupt and leave losses with the DI fund acts like a subsidy to the holder of the bubble asset. Since the payoff to the banker conditional on default is bounded at zero, and deposits are treated as risk free by insured depositors, the implicit subsidy a bubbly bank receives from the DI fund in the event of bubble collapse at time \( t + 1 \) is defined as:

\[ S_t = \max \left( R^d_t d_t^b - R^l_t b_t^b, 0 \right). \]

Then the subsidy per unit of investment in bubbles is:

\[ \rho_t = \frac{S_t}{\mu_t m_t^b} = \frac{1}{\xi} \max \left( \frac{R^d_t}{1 - \lambda} \phi_t - R^l_t (1 - \xi), 0 \right), \]  \hspace{1cm} (34)

\(^\text{12}\)A possibility that we do not focus on in the paper is that \( E_t \left[ (1 - \gamma + \gamma \phi_{t+1}) \left( u_{t+1}^b - u_{t+1}^l \right) \right] > 0 \).

and every single bank invests in the bubble while the supervisory constraint (10) binds. In the Appendix we derive the conditions under which this may happen and provide more discussion of the reasons why we do not consider this to be an economically relevant situation.

\(^\text{13}\)The equalization of the value of lending and bubbly banks means that we can characterize \( \phi_t \) only by examining the safe lending bank.
which is increasing in $\xi$ and $\phi_t$. Equation (34) shows that more leveraged (high $\lambda$ or high $\phi_t$) and less effectively supervised (high $\xi$) banks enjoy a larger subsidy. Therefore the effective time $t+1$ value of bubble holdings for bubbly banks with insured deposits is:

$$\hat{\mu}_{t+1} = \begin{cases} 
\mu_t + 1 & \text{with probability } \pi, \\
\rho_t \mu_t & \text{with probability } 1 - \pi.
\end{cases}$$

After some manipulation we can show that (32) can be re-written as follows:

$$E_t \left[ \left(1 - \gamma + \gamma \phi_{t+1}\right) \left(R_t^l - \frac{\hat{\mu}_{t+1}}{\mu_t}\right) \right] = 0. \quad (35)$$

In other words, the presence of deposit insurance is equivalent to a bailout subsidy which compensates bankers for a fraction $\rho_t$ of losses on their bubble holdings.\(^{14}\)

(35) comes from equalizing the value of bubbly and lending banks but it works essentially like an arbitrage condition between bubbles and loans. It differs in three crucial aspects from the condition for entrepreneurs (17). First of all, the opportunity cost of investment is different: $R_t^d$ for savers and $R_t^l \geq R_t^d$ for banks. Secondly, the state income valuations differ due to the different preferences of bankers and entrepreneurs. But thirdly, the risks faced by the two groups of potential bubble investors are different because of their different access to the financial safety net.

The ability to issue insured deposits creates a subsidy for the bubble holder which only banks can take advantage of. These differences will play an important role in determining the distribution of bubble holdings between banks and savers. We will devote Section 5.2 to analysing this issue.

Finally, it is easy to confirm our initial guesses used in constructing the equilibrium. Lending banks do not want to hold any of the bubble when (22) holds with strict inequality. This is satisfied whenever (35) holds and $\rho_t > 0$. In other words, bubbly and lending banks cannot both hold the bubble if only bubbly banks are subsidized.

The supervisory constraint (10) binds for bubbly banks if $u_{t+1|b} = 0$ and (29) holds. We have

\(^{14}\)When default does not occur, the subsidy disappears ($\rho_t = 0$) and (32) can be written as follows:

$$E_t \left[ \left(1 - \gamma + \gamma \phi_t \right) \left(R_t^l - \frac{\hat{\mu}_{t+1}}{\mu_t}\right) \right] = 0.$$

In other words, the value of returns from loans and bubble holdings is equalised.
\[ u_{t+1|c} = 0 \] as long as debt liabilities are greater than income from loans:

\[ R_t^d \frac{\phi_t}{1 - \lambda + \phi_t} \geq R_t^l (1 - \xi). \] (36)

To gain some intuition about (36) consider the case when \( R_t^d = R_t^l \). Then (36) holds when the share of bubbles in total assets is greater than the share of equity in total liabilities.

\[ \xi \geq \frac{1 - \lambda}{1 - \lambda + \phi_t}. \] (37)

Finally, in an equilibrium where banks’ bubble holdings are positive, equation (29) must hold as long as \( \rho_t < 1 \). Otherwise, no bank will invest in bubbles.

### 3.3 Workers’ optimal behaviour

Workers cannot operate the production technology. Hence they cannot pledge collateral to lenders and cannot borrow. Because ours is a limited commitment economy, we guess and verify that \( R_t^d < \beta^{-1} \) at all times along the equilibrium paths we consider. Consequently workers will save nothing and their labour supply \( h_t \) is given by

\[ h_t = w_t^\eta. \] (38)

### 3.4 Aggregation and market clearing

The bubbles held by unproductive entrepreneurs and banks add up to the total supply of the bubble which we normalize to 1:

\[ m_t^c + m_t^b = 1. \] (39)

Let \( Z_t^H \) and \( Z_t^L \), respectively, denote aggregate wealth of the productive and unproductive entrepreneurs. Then we can characterise the aggregate equilibrium as follows. From (19) the aggregate employment of the productive entrepreneurs is given by

\[ H_t^H = \frac{\beta Z_t^H}{w_t - \theta a_t^H / R_t^l}. \] (40)
When (15) holds, the unproductive entrepreneurs are indifferent between making deposits and producing, thus their aggregate saving is split as follows

\[ w_t H_t^L = \beta Z_t^L - D_t - m_t^e \mu_t, \]  

(41)

where \( D_t \) denotes aggregate deposit.

Aggregating equation (23) and (27) we get an expression for aggregate deposits:

\[ D_t = \frac{\phi_t}{(1 - \lambda)} \gamma N_t. \]  

(42)

\( \gamma N_t \) is the net worth of surviving bankers. The aggregate balance sheet of the banking sector is

\[ D_t + \gamma N_t = B_t + m_t^b \mu_t, \]  

(43)

where \( B_t \) denotes aggregate loans which are given by

\[ B_t = w_t H_t^H - \beta Z_t^H. \]  

(44)

Let us turn to the transition of state variables. Note that the unproductive entrepreneurs become productive in the next period with probability \( n\delta \) and the productive entrepreneurs continues to be productive with probability \( 1 - \delta \). Their rates of return are given by (15) and (18). Therefore the net worth of the productive entrepreneurs evolves from (13) and (18) as

\[ Z_{t+1}^H = (1 - \tau_{t+1}) \left\{ (1 - \delta) \frac{a^H(1 - \theta)}{w_t - \theta a^H / R_t} \beta Z_t^H + n\delta \left[ R_t^d \left( \beta Z_t^L - m_t^e \mu_t \right) + m_t^e \tilde{\mu}_{t+1} \right] \right\}. \]  

(45)

Similarly, the aggregate net worth of the unproductive entrepreneurs evolves as

\[ Z_{t+1}^L = (1 - \tau_{t+1}) \left\{ \delta \frac{a^L(1 - \theta)}{w_t - \theta a^H / R_t} \beta Z_t^L + (1 - n\delta) \left[ R_t^d \left( \beta Z_t^L - m_t^e \mu_t \right) + m_t^e \tilde{\mu}_{t+1} \right] \right\}. \]  

(46)

Aggregating the production function, aggregate output is given by

\[ Y_t = a^H H_{t-1}^H + a^L H_{t-1}^L. \]  

(47)
Aggregating bankers’ net worth evolution equation we have:

\[ N_{t+1} = \gamma \left[ R^d_t B_t + m^b_t \mu_{t+1} - R^d_t D_t \right]. \]  

(48)

Finally the government’s budget constraint implies that taxes will be levied on entrepreneurs in order to bail out the depositors in bubbly banks. This implies that

\[ \tau_{t+1} = \begin{cases} 0 & \text{with probability } \pi, \\ \rho_t m^b_t \mu_t & \text{with probability } 1 - \pi, \end{cases} \]  

(49)

where \( m^b_t \mu_t \) is the value of the banking sector’s bubble purchase in period \( t \) and \( \rho_t \) is given by (34). The tax rate is zero whenever the bubble survives and no bailout is needed.

The markets for goods, labour, capital, loans and deposits must clear. Goods market clearing implies that aggregate saving must equal to aggregate investment:

\[ \beta (Z^H_{t+1} + Z^L_t) + \gamma N_t = w_t (H^H_t + H^L_t) + \mu_t. \]  

(50)

From (38), labour market clearing implies

\[ w^\eta_t = H^H_t + H^L_t. \]  

(51)

Equations (16), (17), (33), (34), (35), (39)-(51) jointly determine 17 variables \( R^d_t, R^l_t, w_t, H^H_t, H^L_t, Y_t, \phi_t, D_t, B_t, Z^H_{t+1}, Z^L_{t+1}, N_{t+1}, \mu_t, m^c_t, m^b_t, \tau_t, \rho_t \) with three states \( Z^H_t, Z^L_t, N_t \).

**Definition 1** Competitive bubbly equilibrium is a sequence of decision rules \( \{ H^H_t, H^L_t, Y_t, D_t, B_t, m^c_t, m^b_t \}_{t=0}^\infty \), aggregate state variables \( \{ Z^H_{t+1}, Z^L_{t+1}, N_{t+1} \}_{t=0}^\infty \) and a price sequence \( \{ R^d_t, R^l_t, w_t, \phi_t, \mu_t \}_{t=0}^\infty \) such that: (i) entrepreneurs, banks and workers optimally choose decision rules \( \{ H^H_t, H^L_t, Y_t, D_t, B_t, m^c_t, m^b_t \}_{t=0}^\infty \) taking the evolution of aggregate states, prices and idiosyncratic productivity opportunities as given; (ii) the price sequence \( \{ R^d_t, R^l_t, w_t, \phi_t, \mu_t \}_{t=0}^\infty \) clears the goods, labor, capital, loan, bubble and deposit markets and (iii) government taxes \( \tau_t \) and implicit subsidies \( \rho_t \) satisfies the government budget constraint (49); (iv) the equilibrium evolution of state variables
\( \{ Z_{t+1}^H, Z_{t+1}^L, N_{t+1} \}_{t=0}^{\infty} \) is consistent with the individual choices of entrepreneurs, banks and workers and with the exogenous evolution of productive opportunities at the individual entrepreneur level.

4 Calibration

We have 9 parameters \( \{ \eta, a^H/a^L, \delta, n, \theta, \gamma, \beta, \lambda, \xi \} \). There is little consensus regarding \( \eta \), the Frisch elasticity of labour supply. Micro-data evidence suggests a value close to zero based on the labour supply behavior of primary earners. At the other extreme, Gertler and Kiyotaki (2010) set the Frisch elasticity to 10, justifying their choice by the presence of labour market frictions. We pick a value of \( \eta = 5 \), which is within the range set in calibrating macro models. Following Aoki et al. (2009) we set a value for \( a^H/a^L = 1.1 \). In the baseline, we calibrate the share of bubbles that can be held by an individual bank to \( \xi = 0.05 \). A bank holding a \( \xi \) share of bubbles on its balance sheet would experience a 50% fall in net worth following a bubble collapse but would be solvent.

We calibrate the remaining 6 parameters to match the steady state of the model in the absence of bubbles to 7 moments in US data. These are (1) the real loan rate minus the growth rate of real GDP and minus intermediation costs; (2) the real deposit rate minus the growth rate of real GDP; (3) commercial bank leverage; (4) average corporate leverage; (5) average leverage for highly leveraged corporates; (6) the rate of return on bank equity and (7) the ratio of M2 to GDP.

Calibration targets (1) and (2) deserve further discussion. For simplicity, we assume no growth and no intermediation costs. Growth in the US has averaged close to 3% per annum since the second world war. We are interested in the dynamic efficiency of the investments of US savers and banks which is why we subtract the real growth rate from the real return on deposits and loans.

When we evaluate the dynamic efficiency of banks’ loan investments, we need to take intermediation costs into account. We use FDIC data on US commercial banks’ cash flow sources to measure intermediation costs and we assume that all of these arise due to loan issuance rather than deposit taking. This assumption is reasonable given the labour intensive nature of arranging loans, monitoring them and then recovering them if they become non-performing. We subtract these loan costs from banks’ real loan returns to get the final numbers shown in Table 1. Table 2 presents the values of the parameters chosen to match the moments.

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\(^{16}\)More information on the data sources used in calibration are given in Table A1 in the Appendix.

\(^{17}\)\( \delta = 0.167 \) implies that on average the productive entrepreneur remains productive for six years, and \( n = 0.011 \)
5 Features of Bubbly Equilibrium

5.1 Credit frictions and the existence of bubbly equilibria

We begin by looking at the conditions for bubble existence. This subsection characterises the deposit rate $R^d_t$ and loan rate $R^l_t$ in the steady state without bubbles and discusses when bubbles can circulate. For bubbles to circulate, two conditions are needed. Firstly, bubbles should be attractive. For savers, the opportunity cost of investing in bubbles is the deposit rate, and for banks it is the lending rate. Secondly, bubbles should be affordable. This implies that the rate of return of bubbles conditional on survival is no larger than the rate of economic growth (assumed to be zero).

As a benchmark case here we show the condition for the existence of bubbly equilibria when $\pi = 1$. Binding credit constraints suppress the interest rates below $\beta^{-1}$. Similarly to Farhi and Tirole (2012), when $\pi = 1$ whether a bubbly steady state exists and who owns bubbles depend on whether the two interest rates are lower than the growth rate in the 'no bubbles' steady state.

In our economy, the severity of credit frictions is represented by two parameters, $\lambda$ and $\theta$. Figure 1a shows the region of $\lambda$ and $\theta$ in which the deposit rate is less than one and low productivity agents produce in equilibrium (the red area). In this case, the savers (unproductive entrepreneurs) have an incentive to buy bubbles in order to boost the rate of return they receive on their savings. The blue parts of the graph show parts of the parameter space where the economy is very credit constrained. At such low values of $\lambda$ and $\theta$ low productivity entrepreneurs are active but wages are so low that even such inefficient projects deliver a rate of return greater than unity. As a result, savers have no incentive to hold bubbles in such economies. The white parts of the graph (very high values of $\lambda$ and $\theta$) show parts of the parameter space where low productivity entrepreneurs do not produce because the financial system is well developed. Here again, the rate of return on deposits is greater than unity and savers have no incentive to hold bubbles. So it should be clear from Figure 1b that the conditions for the existence of bubbles is satisfied at intermediate levels of financial development.

The red area of Figure 1b shows the region in which the loan rate is less than one. Then the

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18 See Aoki et al. (2009) for the general discussion of the relationship between the interest rate and credit frictions.
19 In Martin and Ventura (2012) emergence of bubbles itself can creates a "pocket of dynamic inefficiency" so that bubbly equilibria may exist even when the interest rate is greater than the growth rate in no-bubble equilibrium. This result depends on their model setting that credit-constrained agents can create bubbles in subsequent periods. We abstract from new bubble creation. Because of this assumption, the condition for existence of bubbly equilibria reduces to whether the interest rate in a no-bubble equilibrium is lower than the growth rate.
banks have an incentive to buy bubbles. Since the deposit rate is always lower than the loan rate, the savers also have incentive to hold bubbles at these parameter values.

5.2 Who holds the bubble in equilibrium?

Who holds bubbles in equilibrium is an important question for our paper. As we show in subsequent analysis, bubble ownership is key to understanding the macroeconomic effects of boom and bust episodes. This is why we devote this subsection to a thorough analysis of which economic agents own the bubble in equilibrium.

We focus on the stochastic steady state with bubbles. This is a stationary equilibrium in which endogenous variables are constant, including the ratio of bubbles to GDP. When agents in the economy make their decisions, they understand the fact that the bubble can burst in the next period with probability \( 1 - \pi \) and that once it collapses, it will never arise again.

We focus on three determinants of bubble ownership: bank leverage, the risk free real interest rate and the quality of supervision which determines the size of the deposit insurance subsidy to risk-taking. Our choice of these three factors is motivated by actual developments in the 2000s. We provide further discussion of the way our model relates to real world events in Section 5.2.5.

5.2.1 Bubble ownership in the baseline

We start with a baseline version of our model economy in which \( \xi = 0.05 \) and banks’ equity (at just over 9% of assets) is greater than the maximum bubble investment they can undertake without detection. This means that the combination of the banks’ constraints on leverage and on bubble investments is successful at ensuring that all banks remain solvent in the event of bubble collapse \( \left( u_{t+1}|c > 0 \right) \). Hence the DI subsidy for holding bubbles is zero and the supervisory constraint (10) does not bind. Under this calibration we can study the bank’s incentive to hold the bubble when this is neither subsidised nor constrained by government policy.

Table 3 shows how the stochastic steady state and, in particular, the identity of the bubble owner changes as we vary \( \pi \). The first row of Table 3 shows that banks mostly do not hold the bubble for the bursting probabilities we consider. This is because their borrowing constraints bind, leading to a lending spread which makes banks’ investment opportunities \( (R^l) \) superior to savers’ \( (R^d) \). It is natural therefore that savers should be more willing to take on the risk of saving via the bubble.
Banks, in contrast, stick to their traditional lending activities since these are more profitable than holding bubbles. This can be seen from the fourth row of Table 3 which shows that, apart from the $\pi = 0.985$ case, the expected bubble return is substantially below the lending rate. Holding bubbles is not attractive in these cases and banks rationally abstain.

However, since savers are risk-averse and aware that the bubble may burst with a positive probability, they require a risk-premium over the safe deposit rate in order to hold risky bubbles. As we can see from the fifth row of the table, when $\pi = 0.985$, this risk premium peaks at approximately 1.2 pp and pushes the expected return of the bubble higher than the lending rate. At this point, banks buy approximately 1.6% of the bubble and thus share in some of its risks with savers. Even though this is a small fraction of the total bubble, highly leveraged banks still experience a substantial (12.2%) decline in net worth when the bubble collapses.

The second row of Table 3 shows that banks never allocate more than 1.2% of their balance sheets to the bubble despite the fact that its return is higher than the lending rate. This may seem surprising given the bank’s linear period utility. However, the stochastic discount factor of the banks is given by $1 - \gamma + \gamma \phi_{t+1}$ which is not constant. And equation (33) implies that $\phi_{t+1}$ becomes larger when the future spread, $R_{t+1}^d - R_{t+1}^d$, increases.

Due to binding borrowing constraints and time-varying loan spreads, banks behave as if they are risk-averse despite their linear period utility. Similarly to Gertler and Karadi (2011), low bank capital leads to high lending spreads. Hence banks’ marginal value of an extra unit of net worth in the bubble collapse state ($\phi_{t+1|c}$) is high when they choose to hold bubbles. This can be seen in the last row of Table 3. When banks hold bubbles ($\pi = 0.985$), the marginal value when bubbles burst (denoted $\phi_{t+1|c}$ in Table 3), is larger than the marginal value when bubbles survive (denoted $\phi_{t+1|s}$ in Table 3). Consequently banks want to ensure some stability in their net worth.

### 5.2.2 Bubble ownership with low interest rates

Table 4 examines the impact of the safe real interest rate. We increase the value of $\beta$ so that the deposit rate in the bubbleless equilibrium declines by 0.25pp relative to the baseline calibration. The reduction of interest rates expands the size of the bubble at all values of $\pi$. Bank bubble holdings increase too, reaching 2.3% of the bubble stock and bank risk grows too (net worth falls by almost 20% when the bubble bursts in the $\pi = 0.985$ case). Lower interest rates induce banks
to buy some of the bubble by boosting its overall size relative to GDP, thereby increasing the risk premium required by savers in order to hold it. As we explained in section 5.2.1, when the risk premium becomes larger than the lending-deposit rate spread, banks start to find the bubble asset an attractive investment.

5.2.3 Bubble ownership with higher bank leverage

We also consider the impact of higher bank leverage. We implement this in the model by increasing \( \lambda \) so that the ratio of liabilities to net worth for bankers increases to 11 from its baseline value of 10.\(^{20} \) The assumption of no DI subsidy is maintained: even after the increase in leverage, bank failure remains a zero probability event. The results are shown in Table 5. The key message is that now banks hold bubbles for a larger range of values of \( \pi \), their holdings are higher (almost 2% of the stock when \( \pi = 0.985 \)) and bank net worth falls by almost 16% when the bubble bursts.

The impact of leverage on lending spreads is the main reason behind the higher bubble holdings. With bank leverage at 11, \( R_l - R_d \) is around 90bps as compared to 110bps at the baseline calibration with bank leverage at 10. Intuitively, the greater supply of credit under the higher value of \( \lambda \) reduces the scarcity of bank capital in the economy and drives down the rents banks earn in equilibrium. The lower spread in turn reduces the wedge between the opportunity cost of funds for banks and for savers. As a result, bubbles become more attractive for banks and they increase their exposures.

5.2.4 Bubble ownership with weaker supervision

Until this point we considered calibrations of our model in which the financial safety net did not affect banks’ investment decisions because supervision was effective at keeping the bank solvent in all states of the world. In this section we examine the impact of the quality of supervision as captured by \( \xi \) - the maximum share of bubbles that can be held without detection.

Table 6 shows how varying \( \xi \) affects the stochastic steady state and, in particular, the identity of the bubble owner. Given \( \xi \), the equilibrium fraction of the bubbly banks is determined by equation

\(^{20}\)Strictly speaking, a higher \( \lambda \) implies that bankers can divert less deposits for their own private benefit. This relaxes their incentive compatibility constraint, giving them access to higher leverage.

By increasing \( \lambda \) we merely want to implement the increase in bank leverage which occurred in the 2000s. We are not necessarily claiming that moral hazard problems for banks declined over the same period. Since deposit diversion is something that never happens in equilibrium, this way of achieving higher leverage in the banking system is not something that is in itself important for bubble ownership - the central purpose of the exercise.
(32), and this in turn pins down the fraction of the bubble owned by the banking sector. We consider the impact of supervision quality under a riskier bubble (Panel A) and a safer bubble (Panel B).

Panel A ($\pi = 0.975$) shows that banks’ risk-taking significantly increases as the quality of supervision declines ($\xi$ increases) but only when $\xi$ becomes large enough. This follows naturally from the analysis in section 3.2.4. For the DI subsidy to bubble holdings to be positive, $\xi$ needs to be large enough so that the bubbly bank (which invests a $\xi$ fraction of its balance sheet in the bubble) goes bankrupt when the bubble collapses. Once $\xi$ increases sufficiently ($\xi = 0.1$ and higher), banks start to enjoy a substantial deposit insurance subsidy and invest in the bubble. For example, when $\xi = 0.3$ the DI subsidy is worth more than 60% of bubble purchases encouraging around 73% of banks to become ‘bubbly’. They all fail as boom turns to bust.

In Panel B we examine the case of $\pi = 0.985$ (bursts happen once every 67 years). Banks now hold bubbles throughout the range of $\xi$ values we consider, even for cases when the DI subsidy is zero. Reducing the quality of supervision (increasing $\xi$ towards unity) increases the incentive for banks to hold bubbles because it shields them from an increasing fraction of the potential losses. The share of bubbles held by banks increases to 36.9% as $\xi$ rises to 0.3. As the banks’ bubble holdings grow, the banking sector expands to absorb these and take advantage of the government guarantee on its risky bubble holdings. The net worth of the banking system relative to GDP increases from 6% to 8.3% as $\xi$ increases from 0.05 to 0.30.

As banks’ bubble holdings increase, bank risk grows substantially. The penultimate two rows of Table 6 measure the impact on bank balance sheets when the bubble bursts. When $\xi = 0.30$ bank losses reach almost 7% of GDP. Bank capital also experiences much larger falls during the crisis when $\xi = 0.30$, falling by around 75% when the bubble bursts.

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21 The growth of banks’ bubble holdings is not driven by the mechanical effect of higher $\xi$. Even though each individual bank is constrained to a $\xi$ bubble share, the banking system’s exposure to the bubble grows mainly because more bankers choose to set up ‘bubbly’ rather than ‘lending’ banks. The importance of $\xi$ is to increase the DI subsidy ($\rho$) and encourage more banks to operate risky rather than safe banks.

22 Laeven and Valencia (2010) report cross-country bank failure rates in the 2007-09 period. Using a wide definition of bank failure which covers banks that had to resort to substantial government assistance, they put the fraction of failing US banks at over 20% of bank assets while France, Greece, Holland and Ireland are all in the 60-80% range. Iceland is the biggest outlier with 90% actual bank insolvency. The bank failures in our model are not excessive compared to those reported by Laeven and Valencia (2010). When $\xi = 0.15$, 20.7% of banks fail which is very close to the number for the US. When $\xi = 0.3$, 73.4% of banks fail - similar to the experience in a number of European countries.
5.2.5 Discussion

Our numerical results show that financial institutions are not significantly exposed to the bubble asset in a baseline in which interest rates are high, risky assets are small relative to bank capital and bank profitability is high. Motivated by recent history we consider three important factors that may induce banks to invest in the bubble. Weak supervision (captured by a large value of $\xi$) allows the creation of ‘bad banks’ which are highly profitable during booms and then go bankrupt in the bust, leaving losses with taxpayers. The possibility of such ‘bad banks’ encourages the banking system to invest in the bubble asset because its risks are partly underwritten by the financial safety net. Second, we examined the impact of higher bank leverage (higher $\lambda$) and found that it also encourages banks to invest in risky assets through a channel that is reminiscent of the ‘bank charter value’ of Keeley (1990). When bank leverage is low, credit is scarce and lending-deposit spreads are high meaning that banks’ traditional (and safe) lending business is more attractive than holding the bubble asset. Finally, low risk-free interest rates also increase the size of the bubble and hence more of it ends up in the hands of the banks.

The decline of real interest rates over the past 30 years worldwide has been well documented. Krugman (2014) and Summers (2014) (among many others) have both shown evidence that the long term interest rate has fallen from around an average of 5% in the 1980s to 2% in the 1990s and to an average of 1% in the 2000s.

A number of developments in the 2000s conspired to increase financial system leverage. One notable factor was regulatory arbitrage. As documented by Acharya et al. (2013), many commercial banks set up special purpose vehicles (SPVs) that held securitized products the banks themselves had originated. These structures attracted much lower capital requirements compared to holding the loans on-balance-sheet.

Another important factor behind the increase in leverage was the introduction in 2004 of the Basel II system of risk-sensitive capital requirements. The new regime allowed banks to use risk-weights based on their own risk management models. In practice, this made it easier to manipulate capital requirements downwards (Haldane (2013), Calomiris (2009)). The security broker-dealer sector in particular\(^{23}\) used this relaxation in capital requirements to increase its leverage and grow

\(^{23}\)The US delayed the introduction of Basel II and its commercial banks remained on the Basel I standard and could not expand their leverage. However, as Calomiris (2009) documents, US investment banks successfully lobbied to be allowed to use their own risk management models, effectively allowing them to expand their leverage very substantially
its balance sheet from 21% of depository institutions’ assets in 1995 to 35% in 2007.\textsuperscript{24}

The same shifts in the structure of the financial system are also likely to have reduced supervisory effectiveness in the 2000s by shifting more banking activity into less regulated entities such as the SPVs discussed in Acharya et al. (2013) and the broker-dealer sector. Finally, US Flow of Funds data shows that the size of the balance sheets of the GSEs (Fannie Mae, Freddie Mac and others) increased from 16% of depository institutions’ balance sheets in 1995 to 24% in 2007.\textsuperscript{25} Calomiris (2009) attributes this to political pressure on the GSEs to relax their underwriting standards and expand lending in order to promote ‘affordable housing’.

What is crucial for the exercises we perform in our paper is that the financial intermediation sector was able to invest more easily in risky assets but continued to enjoy explicit or implicit state guarantees. The off-balance-sheet vehicles of commercial banks benefited from deposit insurance due to the support of sponsoring banks. The GSEs enjoyed an explicit government guarantee on their liabilities and the systemic nature of broker-dealers meant that they benefited from an implicit guarantee. In the end, these institutions (with Lehman Brothers’ notable exception) were all rescued by the government. This justifies our approach of considering the impact of deposit insurance alongside weaker supervision on the bubble holdings of the financial intermediary sector.

\section*{5.3 Bubble ownership and amplification}

In the previous section we studied the determinants of bubble ownership. The reason why this matters is because the identity of the bubble is very important for the real effects of the bubble during the boom as well as during the bust. This section uses numerical simulations of our model to demonstrate how saver-held and bank-held bubbles affect the real economy.

Figure 2 compares the dynamics of the economy starting at the stochastic steady state and tracking the economy’s evolution after the bubble collapse. We compare two scenarios. In one, the bubble is held mostly by banks due to lax supervision and a high DI subsidy ($\xi = 0.3$). In the other, there is no DI subsidy ($\xi = 0.05$) and the bubble is held by savers. In each case, we choose the probability of bubble survival ($\pi$) so that the value of the bubble is 20% of GDP. Therefore the difference between the two scenarios in Figure 2 is due to bubble ownership rather than bubble size.

\textsuperscript{24}2014 US Flow of Funds Tables L.109 (Depository Institutions) and L.128 (Security Brokers and Dealers).
\textsuperscript{25}2014 US Flow of Funds Tables L.109 (Depository Institutions) and L.123 (GSEs).
The vertical axis of the figure shows the percentage deviation of each variable from its no-bubble steady state.

The main feature of the simulations shown in the figure is that the economy in which banks are more exposed to the bubble experiences a more volatile path for output and net worth during the bubbly episode. While the collapse of the saver-held bubble actually generates an expansion in output\(^{26}\), output falls under the bank bubble. When banks are exposed to the bubble, they lose a large portion of net worth when the bubble collapses, causing a sharp lending contraction and an increase in the lending-deposit rate spread. This results in a credit crunch and pushes down the investment of the productive entrepreneurs.

Table 7 quantifies a number of channels that drive the positive real effects of bubbles during the boom. We use \(\xi\) to generate a range of scenarios (the different columns of the table) with high or low rates of bank bubble ownership. Again, for each value of \(\xi\) we pick a probability of bubble survival (\(\pi\)) to ensure that the aggregate bubble value is equal to 20% of GDP.\(^{27}\)

The table shows that bank-held bubbles (which occur at higher values of \(\xi\)) have larger real effects because they boost aggregate TFP and employment. TFP increases due to a combination of the liquidity effect of Farhi and Tirole (2012) and the investment composition effect of Martin and Ventura (2012).\(^{28}\) The ‘liquidity effect’ occurs because the availability of the bubble increases the rate of return on savings for unproductive entrepreneurs and they get richer as a result. Consequently, the inflow of wealth into the pool of productive agents increases (each unproductive individual switches to being productive with probability \(1 - n\delta\)). This is an important reason why the wealth of the currently productive group increases (row (6) of Table 7). The wealthier high productivity group then absorbs a greater proportion of labour while the low productivity group absorbs less. This boosts TFP through the investment composition effect of Martin and Ventura (2012).\(^{29}\)

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\(^{26}\) Output expands after bubbles collapse in the case of saver-held bubble because the savers increase their own inefficient production. Bursting of bubbles causes shortage of means of savings. Then the savers increase their production because this represents another means of savings.

\(^{27}\) When \(\xi = 0.05, \pi = 0.978\); when \(\xi = 0.15, \pi = 0.976\); when \(\xi = 0.30, \pi = 0.969\).

\(^{28}\) Eisfeldt and Rampini (2006) empirically show that capital reallocation from less to more productive firms is procyclical. They argue that credit frictions can account for this procyclicality.

\(^{29}\) In Martin and Ventura (2011) the expansionary effect of bubbles arises because bubble creation creates collateral for productive entrepreneurs who cannot borrow against their tangible assets due to a moral hazard problem. This allows productive entrepreneurs to expand borrowing and production, increasing aggregate TFP.

Related to the results in our paper, Martin and Ventura (2011) demonstrate that the effects of bubble creation depend on the identity of the bubble creator. When credit constrained productive firms create bubbles, the bubble is expansionary. When unconstrained savers benefit from bubble creation, the bubble is contractionary as in Tirole (1985).
Table 7 shows that when bubbles are in the hands of savers ($\xi = 0.05$) the TFP-enhancing investment composition effect dominates and accounts for more than 90% of the output increase. In contrast, when banks are the main bubble holders ($\xi = 0.30$), more than half of the output increase comes from higher employment.

Bank bubbles generate larger employment effects because of the way lower lending spreads help to offset the negative effect of higher wages on entrepreneurial leverage. High productivity agents’ leverage is given by $1/[w_t (1 - \theta (R^d_t / R^l_t))]$ - it is strongly affected by the real wage rate and lending spreads. A rise in employment (other things equal) boosts wages and increases the strain on high productivity entrepreneurs’ scarce wealth thus putting a brake on the output expansion. This is indeed what happens when savers hold the bubble. Output expands due to the relocation of labour from unproductive to productive agents but total labour is little changed. In contrast, when banks hold the bubble, lending-deposit spreads fall (row (9) of Table 7), allowing aggregate employment (and hence the wage rate) to rise without reducing entrepreneurs’ leverage.

Row (7) and (8) of the table shed some light on the underlying mechanism. Banks’ ownership of the risky bubble asset boosts the rate of return on wealth while the bubble survives. Bubbles carry a risk premium to compensate the holder for the risk of bursting. When no crisis occurs, this risk premium allows the bubble holder to expand its balance sheet. When the bubble holder is a bank, the balance sheet expansion also leads to greater credit supply. Row (8) of Table 7 shows that lending to corporates increases by more when banks are the bubble holders (the $\xi = 0.30$ column) compared to the case when savers are the main holders (the first and second columns). This is the ‘credit supply effect’ and it only works when the bubble is held in the banking system.

This implication of our model framework is in line with the empirical findings of Gilchrist and Zakrajsek (2012) who construct a measure of ‘excess bond premia’ and show that these are highly correlated with bank lending standards, the health of bank balance sheets and economic activity. In our model, loans are risk free and the lending-deposit rate spread is the ‘loan premium’ variable which captures credit supply conditions. As shown in Table 7 and in Figure 2, our model predicts that credit supply is plentiful (and credit spreads are low) during bank-intermediated bubbles through the effect of the bubble on bank balance sheets. Once the bust comes, credit supply contracts and spreads rise exactly in line with the evidence in Gilchrist and Zakrajsek (2012).  

Chakraborty et al. (2014) use the cross-sectional differences in banks’ exposure to the real estate market to identify whether bubbles have crowding in or crowding out effects on real activity and find in favour of the latter at the level of

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30Chakraborty et al. (2014) use the cross-sectional differences in banks’ exposure to the real estate market to identify whether bubbles have crowding in or crowding out effects on real activity and find in favour of the latter at the level of
6 Conclusions

This paper builds a model with explicit financial intermediation in which rational asset price bubbles arise due to credit frictions. We investigate whether banks or ordinary savers own the bubble in equilibrium and study the way the identity of the bubble holder affects the real impact of the bubble on the wider economy.

In the baseline version of our economy, banks have better investment opportunities than savers. As a result, savers have a stronger incentive to ‘search for yield’ by holding bubbles. We examine the factors that may drive the banks to hold the bubble asset. High financial system leverage expands credit supply, drives down lending-deposit spreads, weakening the rate of return advantage of banks over savers and making the former more willing to invest in the bubble asset. Low interest rates expand the size of the bubble, raise the required risk premium for savers and make the bubble more attractive for banks. Finally, government guarantess coupled with weak supervision generate a risk-taking subsidy for banks, encouraging them to take more risk by holding the bubble. We argue that these factors are important in explaining why the financial system became so heavily exposed to the US housing bubble in the 2000s.

The paper then examines the way the bubble’s impact on the real economy depends on who the holder is. When banks are the bubble-holders, this amplifies both the boom while the bubble survives and the bust when it finally bursts. This is because the bubble delivers a high return conditional on not bursting in order to compensate for the tail risk of total loss. During the boom phase, this high return leads to very high profits for the bubble holder. When the banks are the recipients of these profits, this helps to expand credit and boost the net worth of other credit constrained agents. When the savers are the recipients of these profits, no such relaxation of borrowing constraints occurs and the impact of the bubble is relatively limited. The contractionary effect of the bubble collapse is also larger when banks hold bubbles. This is because banks suffer a decline in their net worth and tighten credit supply with negative consequences for real activity.

Our model in which banks are heterogeneous (‘bubbly’ and ‘lending’ banks) is capable of generating such a negative relationship between bank bubble holdings and lending for productive purposes at the level of the individual bank. In ‘bubbly banks’ the greater bubble holding crowds out loans to firms. This implies that our paper is consistent with the evidence presented in Chakraborty et al. (2014).

However, the crowding out effect at the level of the individual bank is more than compensated for by the greater aggregate credit supply effect. As a result, aggregate credit and economic activity expands.
<table>
<thead>
<tr>
<th>Moment (Model concept)</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real deposit rate - real GDP growth ((R^d))</td>
<td>0.950</td>
<td>0.971</td>
</tr>
<tr>
<td>Real loan rate - real GDP growth - costs/Assets ((R^l))</td>
<td>0.982</td>
<td>0.982</td>
</tr>
<tr>
<td>Ratio of M2 to GDP ((D/Y))</td>
<td>0.500</td>
<td>0.464</td>
</tr>
<tr>
<td>Bank leverage ((D/N))</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Average corporate leverage ((L/Z))</td>
<td>0.500</td>
<td>0.530</td>
</tr>
<tr>
<td>Leverage of indebted corporates ((L/(sZ)))</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td>Bank rate of return on equity ((R_t + \frac{\phi_t(R^l_t - R^d_t)}{1-\lambda}))</td>
<td>1.100</td>
<td>1.103</td>
</tr>
<tr>
<td>Parameter</td>
<td>$\delta$</td>
<td>$n$</td>
</tr>
<tr>
<td>-----------</td>
<td>---------</td>
<td>-----</td>
</tr>
<tr>
<td>Value</td>
<td>0.167</td>
<td>0.011</td>
</tr>
</tbody>
</table>
### Table 3: Bubble riskiness and bank risk in the baseline

<table>
<thead>
<tr>
<th></th>
<th>( \pi = 0.975 )</th>
<th>( \pi = 0.980 )</th>
<th>( \pi = 0.985 )</th>
<th>( \pi = 0.99 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of bubble held by banks</td>
<td>0.000</td>
<td>0.000</td>
<td>0.016</td>
<td>0.000</td>
</tr>
<tr>
<td>Bank bubbles/Total bank assets</td>
<td>0.000</td>
<td>0.000</td>
<td>0.012</td>
<td>0.000</td>
</tr>
<tr>
<td>Bubble to GDP ratio</td>
<td>0.118</td>
<td>0.296</td>
<td>0.463</td>
<td>0.559</td>
</tr>
<tr>
<td>( \text{E(Bubble Return)} - R^l )</td>
<td>-0.007</td>
<td>-0.002</td>
<td>0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>( \text{E(Bubble Return)} - R^d )</td>
<td>0.005</td>
<td>0.010</td>
<td>0.012</td>
<td>0.010</td>
</tr>
<tr>
<td>Bank Net Worth/GDP (pre-crash)</td>
<td>0.050</td>
<td>0.055</td>
<td>0.060</td>
<td>0.058</td>
</tr>
<tr>
<td>Bank Loss/GDP</td>
<td>0.000</td>
<td>0.000</td>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>% fall in bank Net Worth</td>
<td>0.000</td>
<td>0.000</td>
<td>0.122</td>
<td>0.000</td>
</tr>
<tr>
<td>( \phi_{t+1</td>
<td>c}/\phi_{t+1</td>
<td>s} )</td>
<td>0.985</td>
<td>0.962</td>
</tr>
</tbody>
</table>

**Note:** \( \phi_{t+1|c} (\phi_{t+1|s}) \) stands for the shadow value of the bank net worth when the bubble collapses (survives) at time \( t + 1 \).
Table 4: Interest rates and bank risk ($\beta = 0.960$)

<table>
<thead>
<tr>
<th></th>
<th>$\pi = 0.975$</th>
<th>$\pi = 0.980$</th>
<th>$\pi = 0.985$</th>
<th>$\pi = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of bubble held by banks</td>
<td>0.000</td>
<td>0.023</td>
<td>0.023</td>
<td>0.008</td>
</tr>
<tr>
<td>Bubble to GDP ratio</td>
<td>0.182</td>
<td>0.387</td>
<td>0.499</td>
<td>0.602</td>
</tr>
<tr>
<td>$\text{E}(\text{Bubble Return}) - \bar{R}^t$</td>
<td>-0.004</td>
<td>0.002</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td>$\text{E}(\text{Bubble Return}) - \bar{R}^s$</td>
<td>0.007</td>
<td>0.013</td>
<td>0.012</td>
<td>0.010</td>
</tr>
<tr>
<td>Bank Net Worth/GDP (pre-crash)</td>
<td>0.050</td>
<td>0.058</td>
<td>0.058</td>
<td>0.056</td>
</tr>
<tr>
<td>Bank Loss/GDP</td>
<td>0.000</td>
<td>0.009</td>
<td>0.011</td>
<td>0.005</td>
</tr>
<tr>
<td>% fall in bank Net Worth</td>
<td>0.000</td>
<td>0.157</td>
<td>0.194</td>
<td>0.088</td>
</tr>
<tr>
<td>$\phi_{t+1</td>
<td>c}/\phi_{t+1</td>
<td>s}$</td>
<td>0.977</td>
<td>1.096</td>
</tr>
</tbody>
</table>

Note: $\phi_{t+1|c}$ ($\phi_{t+1|s}$) stands for the shadow value of the bank net worth when the bubble collapses (survives) at time $t + 1$. 
<table>
<thead>
<tr>
<th></th>
<th>$\pi = 0.975$</th>
<th>$\pi = 0.980$</th>
<th>$\pi = 0.985$</th>
<th>$\pi = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of bubble held by banks</td>
<td>0.000</td>
<td>0.019</td>
<td>0.019</td>
<td>0.004</td>
</tr>
<tr>
<td>Bubble to GDP ratio</td>
<td>0.158</td>
<td>0.354</td>
<td>0.458</td>
<td>0.554</td>
</tr>
<tr>
<td>E(Bubble Return) - $R^b_t$</td>
<td>-0.004</td>
<td>0.001</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td>E(Bubble Return) - $R^d_t$</td>
<td>0.006</td>
<td>0.012</td>
<td>0.011</td>
<td>0.010</td>
</tr>
<tr>
<td>Bank Net Worth/GDP (pre-crash)</td>
<td>0.048</td>
<td>0.055</td>
<td>0.055</td>
<td>0.053</td>
</tr>
<tr>
<td>Bank Loss/GDP</td>
<td>0.000</td>
<td>0.007</td>
<td>0.009</td>
<td>0.002</td>
</tr>
<tr>
<td>% fall in bank Net Worth</td>
<td>0.000</td>
<td>0.126</td>
<td>0.159</td>
<td>0.046</td>
</tr>
<tr>
<td>$\phi_{t+1</td>
<td>c} / \phi_{t+1</td>
<td>s}$</td>
<td>0.981</td>
<td>1.071</td>
</tr>
</tbody>
</table>

Note: $\phi_{t+1|c}$ ($\phi_{t+1|s}$) stands for the shadow value of the bank net worth when the bubble collapses (survives) at time $t + 1$. 
Table 6: Supervision quality and bank risk

<table>
<thead>
<tr>
<th></th>
<th>Panel A: $\pi = 0.975$</th>
<th></th>
<th>Panel B: $\pi = 0.985$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\xi = 0.05$</td>
<td>$\xi = 0.15$</td>
<td>$\xi = 0.30$</td>
<td>$\xi = 0.05$</td>
</tr>
<tr>
<td>Fraction of bubble held by banks</td>
<td>0.000</td>
<td>0.122</td>
<td>0.528</td>
<td>0.016</td>
</tr>
<tr>
<td>Bank bubbles/Total bank assets</td>
<td>0.000</td>
<td>0.031</td>
<td>0.220</td>
<td>0.012</td>
</tr>
<tr>
<td>Fraction of banks holding bubbles</td>
<td>0.000</td>
<td>0.207</td>
<td>0.734</td>
<td>1.000</td>
</tr>
<tr>
<td>Bubble to GDP ratio</td>
<td>0.118</td>
<td>0.146</td>
<td>0.340</td>
<td>0.463</td>
</tr>
<tr>
<td>$E(Bubble \ Return) - R^d$</td>
<td>-0.007</td>
<td>-0.006</td>
<td>-0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>$E(Bubble \ Return) - R^{dt}$</td>
<td>0.005</td>
<td>0.011</td>
<td>0.006</td>
<td>0.012</td>
</tr>
<tr>
<td>Bank Net Worth/GDP (pre-crash)</td>
<td>0.050</td>
<td>0.055</td>
<td>0.090</td>
<td>0.060</td>
</tr>
<tr>
<td>Bank Loss/GDP</td>
<td>0.000</td>
<td>0.012</td>
<td>0.067</td>
<td>0.007</td>
</tr>
<tr>
<td>% fall in bank Net Worth</td>
<td>0.000</td>
<td>0.212</td>
<td>0.746</td>
<td>0.122</td>
</tr>
<tr>
<td>Deposit insurance subsidy ($\rho$)</td>
<td>0.000</td>
<td>0.348</td>
<td>0.625</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table 7: The real effects of bubbles (percentage point deviation from the bubbles steady state)

<table>
<thead>
<tr>
<th></th>
<th>$\xi = 0.05$</th>
<th>$\xi = 0.15$</th>
<th>$\xi = 0.30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Bank bubble holdings (fraction of tot. bubble value)</td>
<td>0.00</td>
<td>12.2</td>
<td>75.6</td>
</tr>
<tr>
<td>(2) % increase in output relative to 'no bubble' SS</td>
<td>1.04</td>
<td>1.38</td>
<td>3.51</td>
</tr>
<tr>
<td>(3) % increase in aggregate employment</td>
<td>0.10</td>
<td>0.35</td>
<td>1.98</td>
</tr>
<tr>
<td>(4) % increase in TFP</td>
<td>0.94</td>
<td>1.03</td>
<td>1.53</td>
</tr>
<tr>
<td>(5) Increase in productive firms’ employment share (pp)</td>
<td>10.1</td>
<td>11.0</td>
<td>16.2</td>
</tr>
<tr>
<td>(6) Productive entrepreneur net worth (% increase)</td>
<td>13.1</td>
<td>14.5</td>
<td>23.1</td>
</tr>
<tr>
<td>(7) Bank net worth (% increase)</td>
<td>14.5</td>
<td>25.9</td>
<td>91.1</td>
</tr>
<tr>
<td>(8) Bank lending to entrepreneurs (% increase)</td>
<td>13.1</td>
<td>14.5</td>
<td>23.8</td>
</tr>
<tr>
<td>(9) Lending-Deposit spread (percentage points)</td>
<td>0.02</td>
<td>-0.01</td>
<td>-0.19</td>
</tr>
</tbody>
</table>
Appendix (Not for publication)

A Bubbles as Collateral

We examine whether entrepreneurs buy bubbles in order to use them as collateral for productive projects. We compute the value in terms of utility of buying a bubble at price $\mu_t$, borrowing against its value the maximum amount possible ($\theta E_t \bar{\mu}_{t+1} / R_t^l$) and then using this borrowed amount as downpayment for a leveraged productive investment. Entrepreneurs will prefer this strategy to simply undertaking a leveraged productive investment if the following condition holds:

\[ E_t \left[ (1 - \tau_{t+1}) \left( \frac{a^H(1 - \theta)}{w_t - \theta a^H / R_t^l} \frac{\theta E_t \bar{\mu}_{t+1}}{R_t^l} + \frac{\bar{\mu}_{t+1} - \theta E_t \bar{\mu}_{t+1}}{\mu_t} \right) \frac{1}{c_{t+1}^H} \right] \geq \frac{a^H(1 - \theta)}{w_t - \theta a^H / R_t^l} E_t \left( \frac{1}{c_{t+1}^H} (1 - \tau_{t+1}) \right). \]  
(A.1)

In general, when $a^H/a^L$ is relatively high, (A.1) will not hold and productive entrepreneurs will not find it profitable to hold bubbles in order to use them as collateral. The intuition for this result is that, since $\theta < 1$, the collateral constraint forces the leveraged holder to use some of his/her own funds in purchasing the bubble. This is costly for productive agents because it reduces the amount they can invest in their very high yielding productive projects. When $a^H/a^L$ is high and $\theta$ is sufficiently below unity, this cost is too high and productive agents do not hold bubbles. In our analysis presented in the main text, we first guess that this will be the case and verify it in any numerical solutions of the model.

In the steady state with safe bubbles ($\pi = 1$), this can be verified analytically. In the deterministic steady state, $\tau_t = 0$, and $\bar{\mu}_{t+1}/\mu_t = 1$ at all times. Therefore (A.1) reduces to

\[ \frac{a^H(1 - \theta)}{w_t - \theta a^H / R_t^l} \frac{\theta}{R_t^l} + (1 - \theta) \geq \frac{a^H(1 - \theta)}{w_t - \theta a^H / R_t^l}. \]  
(A.2)

However, since $R^d > R^d = 1$, it is easily shown that (A.2) is violated. Therefore the productive agents do not invest in bubbles.

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31Here $c_{t+1}^H$ is the consumption of the productive entrepreneur. It is given by

\[ 1/c_{t+1}^H = (1 - \beta)Z_{t+1}^H \]

where $Z_{t+1}^H$ is given by equation (45).
The case in which only bubbly banks operate

In this Appendix, we characterize the case in which only bubbly banks operate in equilibrium. All banks choose to be bubbly banks when $\phi^b_t > \phi^l_t$. This implies

$$E_t \left[ (1 - \gamma + \gamma \phi_{t+1}) u^b_{t+1} \right] > E_t \left[ (1 - \gamma + \gamma \phi_{t+1}) u^l_{t+1} \right]. \quad (B.3)$$

By substituting (24) and (28) into (B.3), we obtain

$$E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) \max \left\{ \left( 1 - \xi \right) R^l_t + \xi \tilde{\mu}_{t+1} \mu_t + \left( 1 - \xi \right) R^l_t + \xi \tilde{\mu}_{t+1} - R^d_t \frac{\phi_t}{1 - \lambda}, 0 \right\} \right]$$

$$> E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) \left\{ R^l_t + \left( R^l_t - R^d_t \right) \frac{\phi_t}{1 - \lambda} \right\} \right]. \quad (B.4)$$

Firstly, consider the case in which the bubbly banks do not default in the event of the bubble collapse. Then equation (B.4) is written as

$$E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) \left\{ R^l_t + \left( R^l_t - R^d_t \right) \frac{\phi_t}{1 - \lambda} \right\} \right]$$

$$> E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) \left\{ \left( 1 - \gamma \right) R^l_t + \xi \tilde{\mu}_{t+1} - R^d_t \frac{\phi_t}{1 - \lambda} \left( 1 - \xi \right) R^l_t + \xi \tilde{\mu}_{t+1} - R^d_t \right\} \right]. \quad (B.5)$$

By arranging terms, (B.5) reduces to

$$E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) \frac{\tilde{\mu}_{t+1}}{\mu_t} \right] > E_t \left[ \left( 1 - \gamma + \gamma \phi_{t+1} \right) R^l_t \right]. \quad (B.6)$$

All banks become bubbly banks in the absence of default when the above equation holds. This condition means that the value of investing in the bubble is higher than investing in loans due to the binding regulatory constraint (10). When this constraint is tight enough and banks want to invest in the bubble even without a DI subsidy, (10) may limit the banking system’s aggregate investment in the bubble asset. Every bank invests in bubbles up to the regulatory limit.

Secondly, consider the case in which the bubbly banks do default in the event of the bubble collapse. It is easy to see that such a situation cannot be an equilibrium. If all the banks are bubbly banks who default in the event of bubble collapse at time $t + 1$, loan supply would become zero at time $t + 1$. Then the shadow value of bank net worth ($\phi^b_{t+1} | c$) becomes very large and this gives
banks strong incentive to maintain some net worth in the state of bubble collapse. Therefore, it is
not optimal for all the banks to become bubbly banks and default when the bubble collapses.

To summarize, there exists an equilibrium in which all banks become bubbly banks, but do not
default even if the bubble collapses. However, in the numerical simulation in Section 5, this type
of equilibrium does not emerge under the calibrated parameters.

C Deriving the Deposit Insurance Subsidy

The binding supervisory constraint implies that

$$b_t^b = \frac{1 - \xi}{\xi} \mu_t m_t^b. \tag{C.7}$$

This means that we can re-write deposits as a function of bubble holdings.

$$d_t^b = \frac{\phi_t}{(1 - \lambda + \phi_t) \xi} \mu_t m_t^b. \tag{C.8}$$

Since the payoff to shareholders conditional on default is zero, the implicit subsidy the bank receives
from the deposit insurance in the event of bubble collapse at time $t + 1$ is defined as:

$$S_t = \max \left( R^d d_t^b - R^d b_t^b, 0 \right)$$

Using (C.7) and (C.8) we have:

$$S_t = \max \left( R^d \frac{\phi_t}{(1 - \lambda + \phi_t) \xi} - R^d \frac{1 - \xi}{\xi}, 0 \right) \mu_t m_t^b$$

which means that the subsidy per unit value of bubble holding is give by:

$$\rho_t = \frac{S_t}{\mu_t m_t} = \frac{1}{\xi} \max \left( R^d \frac{\phi_t}{(1 - \lambda + \phi_t)} - R^d (1 - \xi), 0 \right),$$

which is equation (34) in Section 3.2.
# D Data Sources

Table A1 below outlines the data sources used in calibrating the model.

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<td>Real deposit rate</td>
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Figure 1a. Deposit rate less than one (red area)
Figure 1b. Lending rate less than one (red area)
Figure 2: Comparing a bank-held (solid line) and a saver-held (dashed line) bubble